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DYNAMICS OF THE LOCAL INHOMOGENEITIES OF THE SOLID-PHASE CONCENTRATION IN A FLUIDIZED BED OF MAGNETIC PARTICLES†

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Unsteady problems of the motion and evolution of density inhomogeneities in a developed fluidized bed of ferromagnetic solid particles are considered. The bed is subject to the action of a specified external magnetic field. The inhomogeneities are located in regions where the particle concentration differs from its mean value throughout the bed. The model of the inhomogeneity, proposed previously in [1–3], as a spherical cluster of particles whose boundary is the surface of discontinuity of the distribution of the solid-phase concentration is developed. Both the case of a varying density of the surface of discontinuity, impermeable to solid-phase particles (a constant-mass cluster), and the case of a discontinuity with inflow or outflow of dispersed particles are studied. The effect of the external magnetic field and magnetic properties of the particles on the velocity of motion of inhomogeneities through the bed, their steady-state sizes and lifetimes is evaluated.

THE EFFECT of stabilization of magnetic-particle fluidized beds (with respect to the formation of bubbles) under the action of an external magnetic field has been studied previously experimentally [4, 6] and analytically [7]. Problems of the dynamics of inhomogeneities in non-magnetic particle beds were analysed in [2, 3].

1. STATEMENT OF THE PROBLEM. THE MODEL OF THE SOLID PHASE

Within the framework of the mechanics of continua, the incompressible fluidizing agent and solid particles suspended in it are simulated by interacting interpenetrating fluids, which serve as phases of the fluidized bed. As in [2, 3], we shall restrict our analysis to the limiting case of ideal continua, i.e. we shall assume that viscosity has no effect on momentum transfer within phases. In this case, the viscosity of the fluidizing agent, which is high at distances of the order of the size of a solid particle, occurs only in the term responsible for the interphase friction force. After local averaging of the equation of motion [8] this friction force will assume the form of the external volume force acting on each phase from the other one.

In the case of gas fluidization, when the gas to particle density ratio d_f/d_s is negligible, this force can be approximated by a linear function of the interphase slip velocity [9]. Henceforth the subscripts f and s indicate the fluidizing agent and the solid particles, but after averaging they mean the fluid and solid phases, respectively.

Assume that the fluid phase has no magnetic properties and is not affected by the external magnetic field applied to the system. Let the material of the solid particles be ferromagnetic, and let its magnetization in the external field be reversible (the effects of hysteresis and

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residual magnetization in the solid phase at neither the macroscale nor the microscale level are considered). This means that the magnetic induction in the fluidized system is a single-valued function of the magnetic field strength.

We will restrict our analysis to the case when the dispersed particles and the fluidizing agent have no conduction currents. Then the locally averaged equations of the field in the double continuum have the following form

$$\operatorname{rot}\mathbf{H} = 0, \quad \operatorname{div}\mathbf{B} = 0 \tag{1.1}$$

Here **B** and **H** are the induction and intensity of the magnetic field, respectively.

Taking into account the above assumptions regarding the properties of the particle material, the volume density of the ponderomotive force in the solid phase in a frame of reference associated with a cluster can be represented (in the absolute system of units) in the form [10]

$$\mathbf{F}_m = -\nabla\left\{\frac{\mathbf{H}^2}{8\pi} + \int_0^{\mathbf{H}} \left[\mathbf{M} - d\left(\frac{\partial \mathbf{M}}{\partial d}\right)_{T,\mathbf{H}}\right] d\mathbf{H}\right\} + \frac{1}{4\pi}(\mathbf{B}\nabla)\mathbf{H} \tag{1.2}$$

Here $\mathbf{M} = (4\pi)^{-1}(\mathbf{B} - \mathbf{H})$ is the magnetization vector of the solid phase, $d = \rho d_s$ is its average density, ρ is its volume concentration (i.e. the averaged volume concentration of the dispersed particles) and the derivative $(\partial\mathbf{M}/\partial d)_{T,\mathbf{H}}$ is computed at constant temperature T and magnetic field strength **H**.

We also introduce the standard assumption [10, 11], that the magnetization of the solid phase is proportional to the number of magnetic particles per unit volume. For the relation (1.2) this means that $\mathbf{M} - d(\partial\mathbf{M}/\partial d)_{T,\mathbf{H}} = 0$, and as a result, this relation takes the form

$$\mathbf{F}_m = -\nabla\left(\frac{\mathbf{H}^2}{8\pi}\right) + \frac{1}{4\pi}(\mathbf{B}\nabla)\mathbf{H}$$

In view of the fact that the vector **H** is a potential vector (cf. the first equation in (1.1)) we can write the last equation in the form

$$\mathbf{F}_m = -\frac{1}{4\pi}(\mathbf{H}\nabla)\mathbf{H} + \frac{1}{4\pi}(\mathbf{B}\nabla)\mathbf{H} = (\mathbf{M}\nabla)\mathbf{H} \tag{1.3}$$

We will approximate the dependence of the solid phase magnetization on the magnetic field strength by means of the linear relation $\mathbf{M} = \kappa\mathbf{H}$, where κ is the effective magnetic susceptibility of the solid phase, independent of **H**. Then the magnetic field strength and the induction are also related by a linear law

$$\mathbf{B} = 4\pi\mathbf{M} + \mathbf{H} = (1 + 4\pi\kappa)\mathbf{H} = \mu\mathbf{H} \tag{1.4}$$

where μ is the effective magnetic permeability of the solid phase. Under the last assumption, the expression for the volume density of the ponderomotive force can be modified as follows:

$$\mathbf{F}_m = \kappa(\mathbf{H}\nabla)\mathbf{H} = \kappa\nabla\left(\frac{\mathbf{H}^2}{2}\right) = \frac{\mu - 1}{8\pi}\nabla\mathbf{H}^2 \tag{1.5}$$

The presence of the term (1.5) in the equation of motion of the solid phase is the main difference between the model considered and the models studied previously [2, 3].

Let us introduce the non-inertial frame of reference S with spherical coordinates (r, θ, φ) associated with the centre of a spherical inhomogeneity (cluster) of radius $a(t)$ moving through the bed. The polar axis is directed along the vector of gravitational acceleration **g** (Fig. 1). The cluster velocity in the laboratory frame of reference L is denoted by $\mathbf{U}_d(t)$.

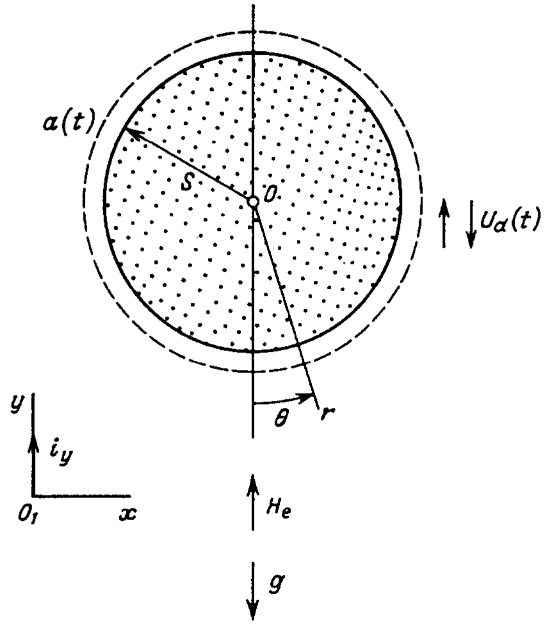

FIG. 1.

Since the relative velocities of the frames of reference S and L are negligible compared to the speed of light c ($U_d/c \ll 1$), we shall ignore relativistic effects and not distinguish between the parameters of the magnetic fields in these frames of reference. For the same reason we adopt relations (1.4) and (1.5) for the function $\mathbf{B}(\mathbf{H})$ and the volume density of the ponderomotive force in the frame of reference S.

Let the disperse system under consideration be subject to the action of the uniform external magnetic field $\mathbf{H}_e$ antiparallel to the gravitational acceleration $\mathbf{g}$ (Fig. 1). Under the action of the field $\mathbf{H}_e$ in the two-phase continuum a resultant field $\mathbf{H}$ is generated. Away from the cluster, where the bed is assumed to be quasihomogeneous, this field is uniform and equal to

$$\mathbf{H}|_\infty = \mathbf{H}_\infty \tag{1.6}$$

Under the assumptions made, we shall formulate the equations of motion and continuity for both phases taking account of the magnetic field for the two simplest models of the inhomogeneity evolution considered in [2, 3].

Model A. A constant mass cluster whose surface is impermeable to the dispersed particles moving through the surrounding bed of constant volume concentration of particles [2]. For this model the jump in the concentration of the solid phase at the cluster boundary is not constant and increases (decreases) during the collapse (expansion) of the inhomogeneity (in this case, the total mass of the cluster is constant). We have

$$\begin{aligned}
&r > a(t), \quad \mathbf{v}(\mathbf{r},t) - \mathbf{w}(\mathbf{r},t) = -k(\epsilon)\nabla p_f(\mathbf{r},t), \quad \nabla \mathbf{v}(\mathbf{r},t) = 0 \\
&d_s\rho \frac{d\mathbf{w}(\mathbf{r},t)}{dt} = -\nabla[p_f(\mathbf{r},t) + p_s(\mathbf{r},t)] - d_s\rho \dot{\mathbf{U}}_d(t) + \frac{1}{8\pi}\nabla[(\mu-1)\mathbf{H}^2(\mathbf{r},t)] + \\
&+ d_s\rho\mathbf{g}, \quad \nabla\mathbf{w}(\mathbf{r},t) = 0 \\
&\epsilon + \rho = 1, \quad \rho = \text{const} \\
&r < a(t), \quad \mathbf{v}'(\mathbf{r},t) - \mathbf{w}'(\mathbf{r},t) = -k'[\epsilon'(t)]\nabla p_f'(\mathbf{r},t) \\
&\partial\epsilon'(t)/\partial t + \epsilon'(t)\nabla\mathbf{v}'(\mathbf{r},t) = 0
\end{aligned} \tag{1.7}$$

$$d_s\rho'(t)\frac{d\mathbf{w}'(\mathbf{r},t)}{dt} = -\nabla[p_f'(\mathbf{r},t) + p_s'(\mathbf{r},t)] - d_s\rho'(t)\dot{\mathbf{U}}_d(t) +$$
$$+\frac{1}{8\pi}\nabla\{[\mu'(t) - 1]\mathbf{H}'^2(\mathbf{r},t)\} + d_s\rho'(t)\mathbf{g}$$
$$\partial\rho'(t)/\partial t + \rho'(t)\nabla\mathbf{w}'(\mathbf{r},t) = 0, \qquad \epsilon'(t) + \rho'(t) = 1$$

Model B. A cluster of constant volume concentration of particles in the homogeneous surrounding bed when there is a particle exchange between the cluster and the bed [3]. In this case, the jump in the concentration at the discontinuity is constant, and the cluster mass increases (decreases) due to the influx of the solid phase to the cluster from outside (its efflux into the surrounding bed). We have

$$\begin{aligned}
&r > a(t), \quad \mathbf{v}(\mathbf{r},t) - \mathbf{w}(\mathbf{r},t) = -k(\epsilon)\nabla p_f(\mathbf{r},t), \quad \nabla\mathbf{v}(\mathbf{r},t) = 0\\
&d_s\rho\frac{d\mathbf{w}(\mathbf{r},t)}{dt} = -\nabla[p_f(\mathbf{r},t) + p_s(\mathbf{r},t)] - d_s\rho\dot{\mathbf{U}}_d(t) +\\
&+\frac{1}{8\pi}\nabla[(\mu - 1)\mathbf{H}^2(\mathbf{r},t)] + d_s\rho\mathbf{g}, \quad \nabla\mathbf{w}(\mathbf{r},t) = 0\\
&\epsilon + \rho = 1, \quad \rho = \text{const}\\
&r < a(t), \quad \mathbf{v}'(\mathbf{r},t) - \mathbf{w}'(\mathbf{r},t) = -k'(\epsilon')\nabla p_f'(\mathbf{r},t), \quad \nabla\mathbf{v}'(\mathbf{r},t) = 0\\
&d_s\rho'\frac{d\mathbf{w}'(\mathbf{r},t)}{dt} = -\nabla[p_f'(\mathbf{r},t) + p_s'(\mathbf{r},t)] - d_s\rho'\dot{\mathbf{U}}_d(t) +\\
&+\frac{1}{8\pi}\nabla[(\mu' - 1)\mathbf{H}'^2(\mathbf{r},t)] + d_s\rho'\mathbf{g}, \quad \nabla\mathbf{w}'(\mathbf{r},t) = 0\\
&\epsilon' + \rho' = 1, \quad \rho' = \text{const}
\end{aligned} \tag{1.8}$$

In relations (1.7) and (1.8) $\mathbf{v}$, $\mathbf{w}$ and p_f, p_s are the locally averaged velocities and pressures of the fluidizing agent and the dispersed particles, respectively, ε is the volume concentration of the fluid phase and $k(\varepsilon)$ is the permeability of the fluidized bed [9]. The prime denotes parameters of the phases inside the cluster $(r < a(t))$.

In both models the distributions of the solid phase outside the cluster as well as inside it are spatially homogeneous.

Consequently, when writing (1.7) and (1.8) it is assumed that the effective magnetic permeability of the solid phase is also a spatially homogeneous function in the domains $r > a(t)$ and $r < a(t)$. In this case, the density of the ponderomotive force in Eq. (1.5) is a potential vector.

In what follows the approximate integration of Eqs (1.7) and (1.8) is performed assuming that the solid-phase flow field outside the cluster is a potential field and that the physical fields for both phases possess axial symmetry. In both cases A and B the magnetic field is described by Eqs (1.1), which satisfy condition (1.6) far from the cluster. At the cluster surface the routine boundary conditions [12] hold, which, when there is axial symmetry, can be written in the following form

$$H_\theta = H_\theta', \qquad B_r = B_r' \tag{1.9}$$

The boundary conditions at the cluster surface for Eqs (1.7) and (1.8) in the non-inertial frame of reference S are the same as those without the magnetic field except for the condition of the normal-stress balance in the solid phase. The latter is derived by integrating the corresponding equation of motion across the discontinuity [13] taking account of the additional magnetic effect (1.5) of the field on the solid phase.

Therefore for Model A the system of boundary conditions has the form

$$r = a(t),\quad w_r = a^\cdot,\quad w'_r = a^\cdot,\quad \epsilon(v_r - a^\cdot) = \epsilon'(v'_r - a^\cdot)$$
$$p_f = p'_f,\; p'_s - p_s = -2\pi(M'^2_r - M^2_r) = \frac{B^2_r}{4\pi}\left[\frac{1}{\mu'} - \frac{1}{\mu} - \frac{1}{2}\left(\frac{1}{\mu'^2} - \frac{1}{\mu^2}\right)\right] \tag{1.10}$$

When writing the last equality the second condition in (1.9) was used.

For Model B the boundary conditions are as follows:

$$r = a(t),\quad \rho(a^\cdot - w_r) = \rho' a^\cdot,\quad \epsilon(v_r - a^\cdot) = \epsilon'(v'_r - a^\cdot) \tag{1.11}$$
$$p_f = p'_f,\quad p'_s - p_s = d_s[\rho(w_r - a^\cdot)^2 - \rho' a^{\cdot 2}] - 2\pi(M'^2_r - M^2_r) =$$
$$= d_s\rho' a^{\cdot 2}\left(\frac{\rho'}{\rho} - 1\right) + \frac{B^2_r}{4\pi}\left[\frac{1}{\mu'} - \frac{1}{\mu} - \frac{1}{2}\left(\frac{1}{\mu'^2} - \frac{1}{\mu^2}\right)\right]$$

Boundary conditions (1.10) and (1.11) at the cluster surface should be supplemented by the conditions of homogeneity of the solid and fluid phases away from the cluster and by the condition that their velocities are bounded over the entire flow field.

2. THE DISTRIBUTION OF THE MAGNETIC FIELD INSIDE AND OUTSIDE THE CLUSTER

For the problem stated, the magnetostatic equations (1.1) with boundary conditions (1.9) and the condition to be satisfied far from the inhomogeneity (1.6) are integrated irrespective of the equations of motion of the phases and in the same manner for both Models A and B. The magnetic field perturbed by a spherical inhomogeneity of the concentration of dispersed particles is described by the following relations

$$r < a(t),\quad H'_r = \frac{3\mu H_\infty}{2\mu + \mu'}\cos\theta,\quad H'_\theta = -\frac{3\mu H_\infty}{2\mu + \mu'}\sin\theta$$
$$r > a(t),\quad H_r = \left[1 + \frac{2(\mu' - \mu)}{2\mu + \mu'}\frac{a^3(t)}{r^3}\right]H_\infty\cos\theta \tag{2.1}$$
$$H_\theta = \left[-1 + \frac{\mu' - \mu}{2\mu + \mu'}\frac{a^3(t)}{r^3}\right]H_\infty\sin\theta$$

where $H_\infty = -H_{r\infty}|_{\theta=0},\; \mu' = \mu'(t)$ for Model A.

From (2.1) it follows that the field inside the cluster is uniform, and outside it is a superposition of the uniform external field and the dipole field with moment $[(\mu' - \mu)/(2\mu + \mu')]a^3(t)$.

In view of relations (2.1) we can represent the magnetic field distribution over the cluster surface in the form

$$\mathbf{H}^2|_{r=a(t)+0} = \frac{9H^2_\infty(\mu'^2\cos^2\theta + \mu^2\sin^2\theta)}{(2\mu + \mu')^2},\quad \mathbf{H}'^2|_{r=a(t)-0} = \frac{9H^2_\infty\mu^2}{(2\mu + \mu')^2} \tag{2.2}$$

The magnetic field vector is discontinuous at the cluster surface: $\mathbf{H}^2|_{r=a(t)+0} \neq \mathbf{H}'^2|_{r=a(t)-0}$.

Note that, due to the uniformity of the magnetic field inside the cluster, $\mathbf{F}'_m = 0$, $r < a(t)$. Since the magnetic properties of the particles in Eqs (1.7) and (1.8) occur only in the equations of the solid-phase motion in the form of the additional ponderomotive force $\mathbf{F}_m$, the relations, obtained in [2, 3] ($\mathbf{H} = 0$, $\mu_s = 1$) for the phase velocity fields and for the pressures of the fluid and solid phases inside the cluster still hold. The action of the magnetic field results in additional normal stresses in the solid phase only in the outer flow region with respect to the cluster $(r > a(t))$.

Boundary conditions (1.10) and (1.11) for the solid-phase pressure jump at the cluster surface can only be satisfied for the adopted model of magnetic particles locally: in the neighbourhood of the front critical point. According to the Davies–Taylor method [14] it leads to a system of ordinary differential equations approximately describing the evolution of the cluster as it moves through the bed.

We shall consider the corresponding equations for Models A and B separately.

3. THE EQUATION OF CLUSTER EVOLUTION. MODEL A

The potential $\varphi_s(\mathbf{r}, t)$ of the disperse-phase flow outside the cluster has the form

$$\varphi_s(\mathbf{r}, t) = U_d(t)\left[1 + \frac{a^3(t)}{2r^3}\right] r\cos\theta - \frac{a^2(t)\,a^{\cdot}(t)}{r} \tag{3.1}$$

When writing (3.1) we took into account that $\mathbf{U}_d(t) = U_d(t)\mathbf{i}_y$ $(\mathbf{i}_y = -\mathbf{g}/g)$ is the unit vector in the direction of the vertical axis of the system of coordinates L (Fig. 1). Hence, for $U_d(t) > 0$ the cluster rises $(\rho' < \rho)$, and for $U_d(t) < 0$ it sinks $(\rho' > \rho)$.

In this case, the second equation in (1.7) admits of a Cauchy–Lagrange integral

$$d_s\rho\varphi^{\cdot}_s(\mathbf{r}, t) + \tfrac{1}{2} d_s\rho\,\mathbf{w}^2(\mathbf{r}, t) + p_f(\mathbf{r}, t) + p_s(\mathbf{r}, t) - d_s\rho\left[1 + \frac{U^{\cdot}_d(t)}{g}\right](\mathbf{g}, \mathbf{r}) -$$
$$- \frac{1}{8\pi}(\mu - 1)\,\mathbf{H}^2(\mathbf{r}, t) = \Phi(t) \tag{3.2}$$

The function $\Phi(t)$ is to be found from the conditions at an infinite distance from the cluster where the phase parameters and the magnetic field are not perturbed

$$r \to \infty, \quad \mathbf{w}^2 \to \mathbf{U}_d^2 (\mathbf{w} \to -\mathbf{U}_d = U_d\,\mathbf{g}/g)$$
$$p_f \to p_{f\infty}(t) + d_s\rho(\mathbf{g}, \mathbf{r}), \quad p_s \to p_{s\infty}, \quad \mathbf{H} \to \mathbf{H}_\infty$$

($p_{f\infty}(t)$ is the fluid-phase pressure at the level of the fluidized bed which coincides with the cluster equatorial plane $\theta = \pi/2$) at the instant of time t. Then integral (3.2) takes the form

$$d_s\rho\varphi^{\cdot}_s(\mathbf{r}, t) + \tfrac{1}{2} d_s\rho[\mathbf{w}^2(\mathbf{r}, t) - U_d^2(t)] + p_f(\mathbf{r}, t) - p_{f\infty}(t) + p_s(\mathbf{r}, t) -$$
$$- p_{s\infty} - d_s\rho\left[1 + \frac{U^{\cdot}_d(t)}{g}\right](\mathbf{g}, \mathbf{r}) - \frac{\mu - 1}{8\pi}[\mathbf{H}^2(\mathbf{r}, t) - H_\infty^2] = 0 \tag{3.3}$$

Based on relations (3.3), (3.1) and (2.2) the pressure distribution of the solid phase over the outer side of the cluster surface may be written in the form

$$p_s(\mathbf{r}, t)\,|_{r=a(t)+0} = \tfrac{1}{2} d_s\rho[U_d^2(t) - a^{\cdot 2}(t) - \tfrac{9}{4} U_d^2(t)\sin^2\theta] -$$
$$- d_s\rho[\tfrac{3}{2} U^{\cdot}_d(t)\,a(t)\cos\theta + \tfrac{3}{2} U_d(t)\,a^{\cdot}(t)\cos\theta - 2a^{\cdot 2}(t) - a(t)\,a^{\cdot\cdot}(t)] +$$
$$+ p_{f\infty}(t) + p_{s\infty} - p_f(\mathbf{r}, t)\,|_{r=a(t)+0} + d_s\rho\left[1 + \frac{U^{\cdot}_d(t)}{g}\right] g a(t)\cos\theta +$$
$$+ \frac{\mu - 1}{8\pi}\left[\frac{9(\mu'^2\cos^2\theta + \mu^2\sin^2\theta)}{(2\mu + \mu')^2} - 1\right] H_\infty^2 \tag{3.4}$$

The solid-phase flow inside the cluster is a superposition of the spherical Hill's vortex with the parameter $U_d'(t)$ of particle circulation velocity, and of the uniform expansion (compression) with the divergence $\zeta(t)=3a^{\cdot}(t)/a(t)$ [2]. Within the framework of the ideal fluid model such a velocity field occurs when the following condition is satisfied

$$U_d'^{\cdot}(t)a(t) + U_d'(t)a^{\cdot}(t) = [U_d'(t)a(t)]^{\cdot} = 0 \tag{3.5}$$

This condition implies the conservation of the scale $\Gamma=U_d'(t)a(t)$ of the solid-phase circulation inside the cluster. Since the equation of the solid-phase motion inside the cluster does not change when passing from the model of non-magnetic particles [2] to Model A ($\mathbf{F}_m'=0$, $r<a(t)$), condition (3.5) still holds.

The solid-phase pressure distribution over the inner side of the cluster surface has the form [2]

$$\begin{aligned} p_s'(\mathbf{r},t)|_{r=a(t)-0} &= -\frac{a(t)a^{\cdot}(t)}{\epsilon k(\epsilon)} + p_{f\infty}(t) - p_f'(\mathbf{r},t)|_{r=a(t)-0} + \\ &+ d_s\rho'(t)\left\{ [\frac{U_d^{\cdot}(t)}{g} + 1]\, ga(t)\cos\theta - {}^1\!/_2\,[a^{\cdot 2}(t) + {}^9\!/_4\, U_d'^2(t)\sin^2\theta] - \right. \\ &\left. - \frac{a^{\cdot\cdot}(t)a(t) - a^{\cdot 2}(t)}{2} \right\} \end{aligned} \tag{3.6}$$

In the neighbourhood of the front critical point ($\theta=\pi$ for a rising cluster and $\theta=0$ for a sinking cluster) $\cos\theta=\mp1\pm\delta/2+O(\delta^2)$, where $\delta=\sin^2\theta\to0$. We will confine ourselves to satisfying the last boundary condition in (1.10) on the solid-phase pressure jump up to first-order quantities in δ. Taking account of relations (2.1) we obtain the following system of equations approximately describing the cluster evolution

$$\begin{aligned} &-\frac{a(t)a^{\cdot}(t)}{\epsilon k\rho d_s} - \frac{\rho'(t)}{\rho}\,\frac{a^{\cdot\cdot}(t)a(t)}{2} - \frac{3}{2}a^{\cdot 2}(t) - a(t)a^{\cdot\cdot}(t) - \\ &-\frac{9}{4}\,[\frac{\rho'(t)}{\rho}U_d'^2(t) - U_d^2(t)] - \frac{p_{s\infty}}{d_s\rho} - \frac{U_d^2(t)}{2} = \\ &= \frac{H_\infty^2\,[\mu-\mu'(t)]^2}{8\pi\rho d_s[2\mu+\mu'(t)]^2}\left\{1 + 14\mu + \frac{6\mu[\mu'(t)-1]}{\mu-\mu'(t)}\right\} \\ &\pm[\frac{\rho'(t)}{\rho} - 1]\,[1 + \frac{U_d^{\cdot}(t)}{g}]\,ga(t) \pm \frac{3}{2}\,[U_d(t)\,a(t) + U_d(t)\,a^{\cdot}(t)] - \\ &-\frac{9}{4}[\frac{\rho'(t)}{\rho}U_d'^2(t) - U_d^2(t)] = \frac{9}{4}\,\frac{H_\infty^2\,\mu[\mu-\mu'(t)]^2}{\pi\rho d_s[2\mu+\mu'(t)]^2} \\ &U_d'(t)a(t) = \Gamma = \text{const} \end{aligned} \tag{3.7}$$

As the equation of state of the solid phase (the plus sign corresponds to a rising cluster and the minus sign to a sinking one) closing system (3.7), we take the linear approximation of the function $\mu(\rho)$

$$\mu(\rho) = 1 + (\mu_0 - 1)\rho \tag{3.8}$$

where $\mu_0 \gg 1$ is the magnetic permeability of the dispersed particle material.

The approximation (3.8) is a result of the assumption made above that the magnetization $\mathbf{M}$ of the solid phase is a linear function of the volume concentration of particles. This assumption was used when writing the ponderomotive force in the form (1.5).

The system of equations (3.7) and (3.8) has the steady-state solution $U_d = U_{d*}$, $U'_d = U'_{d*}$, $a = a_*$, $\rho' = \rho'_*$, $\mu' = \mu'_* = 1 + (\mu_0 - 1)p'_*$ defined by the equations

$$
\begin{gathered}
-\frac{9}{4}\left(\frac{\rho'_*}{\rho}U'^2_{d*} - U^2_{d*}\right) - \frac{p_{s\infty}}{d_s\rho} - \frac{U^2_{d*}}{2} = \\
= \frac{H^2_\infty(\mu-\mu'_*)^2}{8\pi\rho d_s(2\mu+\mu'_*)^2}\left[1 + 14\mu + \frac{6\mu(\mu'_* - 1)}{\mu - \mu'_*}\right] \\
\pm\left(\frac{\rho'_*}{\rho} - 1\right)ga_* - \frac{9}{4}\left(\frac{\rho'_*}{\rho}U'^2_{d*} - U^2_{d*}\right) = \frac{9}{4}\frac{H^2_\infty\mu(\mu-\mu'_*)^2}{\pi\rho d_s(2\mu+\mu'_*)^2} \\
U'_{d*}a_* = \Gamma
\end{gathered}
\tag{3.9}
$$

For the steady-state cluster velocity we obtain from (3.9)

$$
U^2_{d*} = \frac{4}{7}\left(P + \frac{27M\Gamma^2}{16\pi\rho d_s a^5_*} + \frac{p_{s\infty}}{\rho d_s}\right) \tag{3.10}
$$

Here P is the right-hand side of the first equation in (3.9), and $M = \frac{4}{3}\pi p'_* d_s a^3_*$ is the cluster mass, which is conserved in the course of its motion and evolution in the bed. The requirement for the right-hand side of Eq. (3.10) to be non-negative restricts the possibility of conservation of the size of a dense cluster in the course of its motion (for rarefied clusters $\rho'_* < \rho$, $P > 0$).

Eliminating the parameters U_{d*}, U'_{d*} and ρ'_* from the second equation (3.9) we obtain the following equation for the steady-state size of a cluster

$$
\begin{gathered}
a^{12}_* \mp \frac{9}{7}\frac{p_{s\infty}+p_m}{d_s\rho g}a^{11}_* - \frac{9M}{4\pi\rho d_s(2\mu+1)}a^9_* \mp \frac{27M}{14\pi\rho^2 d^2_s g}\left[p_{s\infty}\frac{\mu-1}{2\mu+1} + \right. \\
\left. + p_m(3\mu-1)\right]a^8_* - \left[\frac{27M^2(\mu^2-1)}{16\pi^2\rho^2 d^2_s(2\mu+1)^2} \pm \frac{27M\Gamma^2}{56\pi\rho d_s g}\right]a^6_* + \\
+ \frac{81M^2}{112\pi^2\rho^3 d^3_s g}\left[\mp p_{s\infty}\frac{(\mu-1)^2}{(2\mu+1)^2} \pm p_m(6\mu-1)\right]a^5_* + \\
+ \frac{27M^2(\mu-1)}{16\pi^2\rho^2 d^2_s}\left[-\frac{M(\mu-1)}{4\pi\rho d_s(2\mu+1)^2} \mp \frac{3\Gamma^2}{7g(2\mu+1)}\right]a^3_* \mp \frac{243M^3\Gamma^2(\mu-1)^2}{896\pi^3\rho^3 d^3_s g(2\mu+1)^2} = \\
= 0, \quad p_m = \frac{(\mu-1)^2 H^2_\infty}{8\pi(2\mu+1)^2}
\end{gathered}
\tag{3.11}
$$

Here p_m is the magnetic part of the effective pressure of the solid phase.

In the limiting case when there are no particles within the inhomogeneity domain, i.e. for $M = 0$, relations (3.10) and (3.11) have the form

$$
U^2_{b*} = \frac{4}{9}a_* g + \frac{8\mu p_m}{\rho d_s}, \qquad a_* = \frac{9}{7}\frac{p_m + p_{s\infty}}{\rho d_s g}
$$

and correspond to the results for bubbles [7].

If there are no magnetic effects in the bed, i.e. if $p_m = 0$, Eq. (3.11) reduces to the equation

$$
a^6_* \mp \frac{9}{7}\frac{p_{s\infty}}{d_s\rho g}a^5_* - \frac{3M}{4\pi\rho d_s}a^3_* \mp \frac{27M\Gamma^2}{56\pi\rho d_s g} = 0
$$

studied in [2], for the steady-state size of a pulsating cluster without a magnetic field. In this case, the formula in [2] for the square of the cluster velocity follows from (3.10).

If the magnetic effect predominates over the effects of interaction between the dispersed particles, their circulation in the cluster and the action of the external mass force field, then Eq. (3.1) degenerates into the equality

$$a_*^3 = {}^3/_4 M/(\pi \rho d_s) \tag{3.12}$$

Here it is taken into account that for ferromagnetic particles, i.e. for $\mu_0 \gg 1$, in intensive fields the terms $\sim p_m \mu$ in Eq. (3.11) predominate over the others.

The result (3.12) implies that the cluster size is considered to be steady when its density is identical with the average density $d = \rho d_s$ of the fluidized bed (in this case $U_{d*} = 0$), i.e. the entire bed is quasihomogeneous. The instability of density inhomogeneities means that they have no steady-state sizes (this is relevant for both rarefied and dense clusters).

These conclusions confirm the stabilizing effect of the magnetic field on a three-dimensional fluidized bed. If the magnetic field is intense enough, the inhomogeneities in the form of particle clusters "lose" the steady-state sizes which they had without the magnetic field. This effect is independent of the direction of the vector $\mathbf{H}_\infty$.

Intermediate cases reflecting the interaction of the parameters $p_{s\infty}$, Γ, M and p_m can be considered by the same method as in [2] by appropriate simplifications of Eq. (3.11).

4. EQUATIONS OF CLUSTER EVOLUTION. MODEL B (THE THREE-DIMENSIONAL CASE)

In this model, we consider the simplest case when there is no motion of the dispersed particles inside a cluster [3]. The cluster dynamics is described by the equations of motion and continuity (1.8) with boundary conditions (1.11).

The potential of the solid-phase flow outside the cluster has the form

$$\varphi_s(\mathbf{r}, t) = U_d(t)\ [\,1 + \frac{a^3(t)}{2r^3}\,]\, r\cos\theta - \frac{a^2(t)a^{\cdot}(t)(1-\lambda)}{r}, \quad \lambda = \frac{\rho'}{\rho} \tag{4.1}$$

Equation (1.8) of the solid-phase motion in the region $r > a(t)$ admits of a Cauchy–Lagrange integral in the form (3.3) due to the identity of phase flows outside the cluster in Models A and B.

Using relations (4.1), (3.3) and (2.2) we obtain the pressure distribution of the solid phase over the outer side of the cluster surface in the form

$$\begin{aligned} p_s(\mathbf{r}, t)\,|_{r=a(t)+0} &= \frac{1}{2} d_s\rho[\,U_d^2(t) - a^{\cdot 2}(t)(1-\lambda)^2 - \frac{9}{4} U_d^2(t)\sin^2\theta\,] - \\ &- d_s\rho[\frac{3}{2} U_d^{\cdot}(t)\, a(t)\cos\theta + \frac{3}{2} U_d(t)\, a^{\cdot}(t)\cos\theta - 2a^{\cdot 2}(t)(1-\lambda) - \\ &- a(t)a^{\cdot\cdot}(t)(1-\lambda)\,] + p_{f\infty}(t) + p_{s\infty} - p_f(\mathbf{r}, t)\,|_{r=a(t)+0} + \\ &+ d_s\rho[\frac{U_d^{\cdot}(t)}{g} + 1\,]\, g a(t)\cos\theta + \frac{\mu-1}{8\pi}[\frac{9(\mu'^2\cos^2\theta + \mu^2\sin^2\theta)}{(2\mu+\mu')^2} - 1]H_\infty^2 \end{aligned} \tag{4.2}$$

This distribution differs from the corresponding distribution (3.4) for Model A only because the solid-phase flow through the cluster surface for Model B is non-zero, so the mass is transported to the cluster from outside or away from the cluster into the surrounding bed.

In view of uniformity of the field $\mathbf{H}'$ inside the cluster, there is no ponderomotive force acting on the solid phase in the region $r<a(t)$. Hence, the distribution of the solid-phase pressure p_s' over the inner side of the cluster surface keeps the same form as in [3].

On the basis of relation (4.2), in the same way as in Sec. 3, we obtain the following system of equations of the evolution of inhomogeneity

$$\begin{gathered}
\frac{(\lambda-1)(3-\lambda)}{2}\dot{a}^2(t)+(\lambda-1)a(t)\ddot{a}(t)-\frac{p_{s\infty}}{d_s\rho}+\frac{7}{4}U_d^2(t)- \\
-\frac{a(t)\dot{a}(t)}{\epsilon k\rho d_s}=\frac{H_\infty^2(\mu-\mu')^2}{8\pi\rho d_s(2\mu+\mu')^2}\left[1+14\mu+\frac{6\mu(\mu'-1)}{\mu-\mu'}\right] \\
\pm\frac{3}{2}U_d(t)\dot{a}(t)\pm\frac{1}{2}a(t)\dot{U}_d(t)(1+2\lambda)+\frac{9}{4}U_d^2(t)= \\
=\mp(\lambda-1)ga(t)+\frac{9}{4}\frac{H_\infty^2\mu(\mu-\mu')^2}{\pi\rho d_s(2\mu+\mu')^2}
\end{gathered} \tag{4.3}$$

System (4.3) has the steady-state solution a_*, U_{d*} which at $\mathbf{H}=0$ and/or $\mu_0=1$ reduces to the solution obtained in [3] for non-magnetic particles.

Taking account of "the equation of state" (3.8), for steady-state values of the cluster velocity and its radius, we obtain

$$\begin{gathered}
U_{d*}^2=\frac{4}{7\rho d_s}\left\{p_{s\infty}+p_0\frac{(\mu-1)^2(1-\lambda)[1-\lambda+2\mu(7-4\lambda)]}{[3+(\mu-1)(2+\lambda)]^2}\right\} \\
a_*=\pm\frac{1}{\lambda-1}\frac{9}{7\rho d_s g}\left\{p_{s\infty}+p_0\frac{(\mu-1)^2(1-\lambda)(1-\lambda+6\mu\lambda)}{[3+(\mu-1)(2+\lambda)]^2}\right\} \\
p_0=H_\infty^2/(8\pi)
\end{gathered} \tag{4.4}$$

Henceforth we shall assume that the volume concentration of the solid phase in the homogeneous region of the bed outside the cluster is fairly high ($\rho\geqslant 0.3$), so that $\mu(\rho)\gg 1$ for $r\geqslant a(t)$.

In this case, relations (4.4) have the form

$$U_{d*}^2=\frac{4p_{s\infty}}{7\rho d_s}\frac{\xi-\xi_1}{\xi},\qquad a_*=a_{*0}\frac{\xi-\xi_2}{\xi} \tag{4.5}$$

Here

$$\begin{gathered}
\xi=\frac{p_{s\infty}}{2\mu p_0},\quad \xi_1=\frac{(\lambda-1)[1-\lambda+2\mu(7-4\lambda)]}{2\mu(2+\lambda)^2} \\
\xi_2=\frac{(\lambda-1)(1-\lambda+6\mu\lambda)}{2\mu(2+\lambda)^2},\qquad a_{*0}=a_*(\mathbf{H}=0)=\frac{\mp 9P_{s\infty}}{7\rho d_s g(\lambda-1)}
\end{gathered}$$

The right-hand sides of relations (4.5) for bubbles and rarefied clusters ($0\leqslant\lambda<1$) are non-negative, and for dense clusters ($1<\lambda\leqslant\rho^{-1}$) they are non-negative under the following conditions

$$\xi\geqslant\xi_1(\lambda,\mu),\quad \xi\geqslant\xi_2(\lambda,\mu) \tag{4.6}$$

In the range of values of the relative density $\lambda\in(1,\ \rho^{-1}]$ the first inequality (4.6) is a corollary of the second one. The corresponding relative location of the curves $\xi = \xi_1$ and $\xi = \xi_2$ is shown in Fig. 2.

Hence, in a fluidized bed of magnetic particles we observe the selective effect of the implied magnetic field on the steady-state parameters of inhomogeneities. The steady-state velocity of motion and size of the local rarefactions of the bed density increase under the action of the magnetic field: for $\lambda \in [0, 1)$ we have $\xi_1 \leq 0, \xi_2 \leq 0$. For particle clusters with a relative density $\lambda \in (1, 7/4]$ the action of the magnetic field leads to retardation of their steady-state motion through the bed, while the dense clusters ($\lambda \in (7/4, \rho^{-1}]$), like the rarefied ones, move with higher velocity.

Depending on the magnitude of the magnetic action on the bed the features of the steady-states of the dense clusters will vary as shown in the diagram in Fig. 3. The threshold field $\mathbf{H}_\infty^*$ exists, so that weaker fields have no effect on the existence of a steady-state size of the dense cluster, although this size is reduced compared with a_*. Stronger fields lead to decay of clusters of highest density (with the relative density $\lambda \in [\lambda_*, \rho^{-1}]$). The mass of such a cluster cannot be conserved any longer in the course of its motion through the bed (the range of variation of the relative density of the cluster when there is no steady-state size is shown shaded in Fig. 3). By increasing the field in such a way that $\xi \equiv 0$ it is possible to prevent the existence of steady-state sizes for practically all the dense clusters.

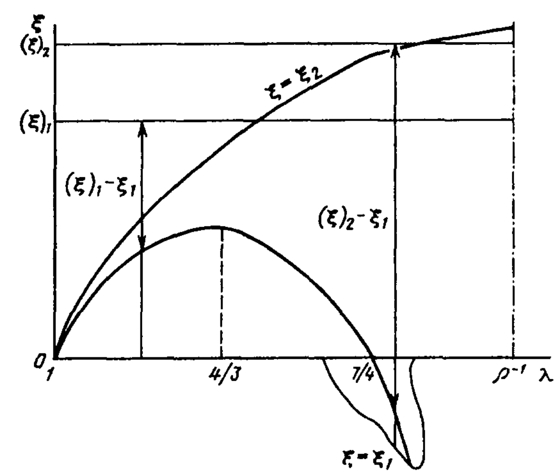

FIG. 2.

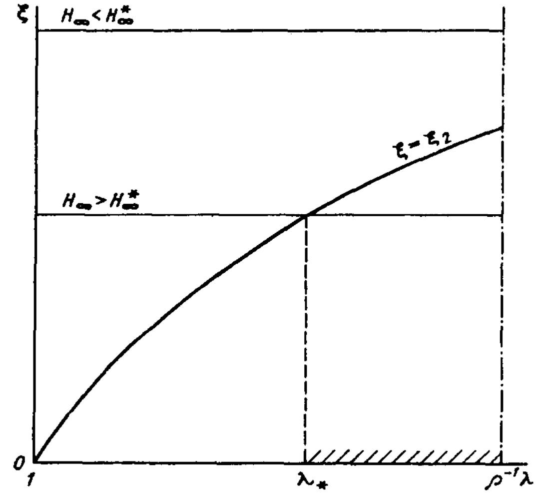

FIG. 3.

The threshold field $\mathbf{H}_\infty^*$ and the lower limit λ_* of values of the cluster relative density, at which a steady-state size of the cluster does not exist, are given by the relations

$$\xi = \xi_2(\rho^{-1}), \quad \xi = \xi_2$$

which yield

$$H_\infty^* = (2\rho + 1)\left[\frac{4\pi p_{s\infty}}{3\mu(1-\rho)}\right]^{1/2}$$
$$\lambda_* = \frac{3 + 2\xi + [(3+2\xi)^2 + 16\xi(3-\xi)]^{1/2}}{2(3-\xi)}, \quad \xi \leqslant \xi_2(\rho^{-1}) \tag{4.7}$$

For Model B we will investigate the stability of the inhomogeneity of the steady-state size to spherically symmetric pulsations of the radius $a(t) = a_* + \Delta(t)$, with the time interval limited by the condition $\Delta(t) \ll a_*$.

Eliminating $U_d(t)$ from system (4.3) we obtain the following differential equation that approximately describes the evolution of inhomogeneity

$$a(t) = \left(\lambda - 4 - \frac{6}{1+2\lambda}\right)\frac{a^{\cdot}(t)a^{\cdot\cdot}(t)}{a(t)} - \frac{7+2\lambda}{(1-\lambda)(1+2\lambda)}\frac{a^{\cdot 2}(t)}{a(t)} -$$
$$- \frac{\eta}{1-\lambda}a^{\cdot\cdot}(t) - \frac{21}{2}\left(1 - \frac{\xi_1}{\xi}\right)\frac{a^{\cdot}(t)}{(1-\lambda)(1+2\lambda)a^2(t)} - \frac{3(3-\lambda)a^{\cdot 3}(t)}{(1+2\lambda)a^2(t)} +$$
$$+ \frac{63}{4(1-\lambda)(1+2\lambda)a^2(t)}\left[\mp(\lambda-1)a(t) - 1 - \frac{4}{7}\eta a(t)a^{\cdot}(t) -\right.$$
$$- \frac{4}{7}(1-\lambda)a(t)a^{\cdot\cdot}(t) - \frac{2}{7}(1-\lambda)(3-\lambda)a^{\cdot 2}(t) + \left.\frac{14\varphi + 2\mu\xi_1}{2\mu\xi}\right] \times$$
$$\times \left[1 + \frac{4}{7}(1-\lambda)a(t)a^{\cdot\cdot}(t) + \frac{2}{7}(1-\lambda)(3-\lambda)a^{\cdot 2}(t) + \frac{4}{7}\eta a(t)a^{\cdot}(t) -\right.$$
$$\left. - \frac{\xi_1}{\xi}\right]^{1/2} \tag{4.8}$$

This equation is written in dimensionless form. In this case, the steady-state size of the bubble $a_{*b} = 9p_{s\infty}/(7\rho d_s g)$ and the quantity $t_0 = \frac{3}{2}(a_{*b}/g)^{1/2}$ serve as the length and time scales, respectively. The parameter η characterizes the relation between the macroscopic (t_0) and microscopic (τ_0) time scales of the transient mode

$$\eta = \frac{1}{18}\frac{C_1\rho}{(1-\rho)^3}\frac{t_0}{\tau_0}, \quad C_1 \approx 150, \quad \tau_0 = \frac{2}{9}\frac{a_p^2 d_s}{d_f \nu_f}$$

(a_p is the radius of the solid-phase particle and ν_f is the kinematic viscosity of the fluidizing agent); $\varphi = \varphi(\mu, \lambda) = \mu(1-\lambda)^2/(2+\lambda)^2$.

In the limiting case $\xi^{-1} \to 0$ $(H_\infty = 0)$, Eq. (4.8) reduces to that obtained in [3].

Equation (4.8) linearized in the neighbourhood of the steady-state size of inhomogeneity leads to the following relation for the small (dimensionless) deviation $\Delta(t)$

$$\Delta^{\cdot\cdot\cdot}(t) + \left[\frac{\eta}{1-\lambda} \mp \frac{9(\lambda-1)}{(1+2\lambda)(1-\xi_2/\xi)}\right]\Delta^{\cdot\cdot}(t) +$$

$$+ \left[\frac{21}{2}\left(1-\frac{\xi_1}{\xi}\right)\frac{1-\lambda}{(1+2\lambda)(1-\xi_2/\xi)} \pm \frac{9\eta}{(1+2\lambda)(1-\xi_2/\xi)}\right]\Delta^{\cdot}(t) \mp$$
$$\mp \frac{63(\lambda-1)^2\Delta(t)}{4(1+2\lambda)(1-\xi_2/\xi)} = 0 \tag{4.9}$$

In this case, in order for the steady-state size to exist it is necessary to satisfy the conditions $\xi > \xi_2 \geqslant \xi_1$ (cf. (4.7)), so that $1-\xi_1/\xi > 0$, $1-\xi_2/\xi > 0$. Consequently, the conclusions [3] concerning the instability of the steady states of any inhomogeneity hold. As in the case when there are no magnetic effects in the bed, among the roots of the characteristic equation for the linear approximation (4.9) there are roots with positive real parts. This indicates the instability of the steady-state size of the cluster to perturbations of the kind considered.

5. EVOLUTION OF A TWO-DIMENSIONAL CIRCULAR CLUSTER. MODEL B

We shall consider the motion and evolution of local inhomogeneities of the solid-phase concentration in a plane fluidized bed of magnetic particles within the framework of the statement made in Sec. 1. The inhomogeneity is modelled by a circular cluster of particles (r and θ are cylindrical coordinates).

The boundary-value problem for the potential of the dispersed-phase external flow has the non-unique solution [3]

$$\varphi_s(\mathbf{r},t) = U_d(t)\left[r + \frac{a^2(t)}{r}\right]\cos\theta + a(t)a^{\cdot}(t)(1-\lambda)\ln\frac{r}{L(t)} \tag{5.1}$$

Here $L(t)$ is an arbitrary function of time with the dimension of length ($L(t) > 0$).

The Cauchy–Lagrange integral, allowed by the equation of solid-phase motion in the region $r > a(t)$, retains the form (3.2). Unlike the three-dimensional case, when defining the function $\Phi(t)$ on the right-hand side of Eq. (3.2), logarithmic perturbations of the pressure fields for both phases far from the cluster are allowed, just as in the case when there was no magnetic field in the model [3].

In this case, the gradients of the pressures p_f and p_s at infinity retain the values they had in the homogeneous bed. Hence, the flow fields of the phases away from the cluster are uniform, just as in the case of the three-dimensional problem.

Making use of relations (3.2) and (5.1) we obtain the distribution of the solid-phase pressure over the outer side of the cluster surface

$$p_s(\mathbf{r},t)\,|_{r=a(t)+0} = \frac{1}{2}d_s\rho\,[U_d^2(t) - (1-\lambda)^2 a^{\cdot 2}(t) - 4U_d^2(t)\sin^2\theta] -$$
$$- d_s\rho\left\{2U_d^{\cdot}(t)a(t)\cos\theta + 2U_d(t)a^{\cdot}(t)\cos\theta + \right.$$
$$\left. + (1-\lambda)[a^{\cdot 2}(t) + a(t)a^{\cdot\cdot}(t)]\ln\frac{a(t)}{L(t)} - (1-\lambda)\frac{a(t)a^{\cdot}(t)L^{\cdot}(t)}{L(t)}\right\} +$$
$$+ p_{f\infty}(t) + p_{s\infty} - p_f(\mathbf{r},t)\,|_{r=a(t)+0} + d_s\rho\left[\frac{U_d^{\cdot}(t)}{g} + 1\right]ga(t)\cos\theta +$$
$$+ \frac{(\mu-1)H_\infty^2}{8\pi}\left[\frac{4(\mu'^2\cos^2\theta + \mu^2\sin^2\theta)}{(\mu+\mu')^2} - 1\right] \tag{5.2}$$

In writing relation (5.2), we used the distribution of the magnetic field over the outer side of the cluster surface in the form

$$\mathbf{H}^2 \mid_{r=a(t)+0} = 4H^2_\infty \frac{\mu'^2 \cos^2\theta + \mu^2 \sin^2\theta}{(\mu + \mu')^2}$$

This distribution was found from the solution of the plane magnetostatic problem.

The solid-phase pressure is distributed over the inner side of the cluster surface in accordance with the expression

$$p_s'(\mathbf{r}, t) \mid_{r=a(t)-0} = -p_f'(\mathbf{r}, t) \mid_{r=a(t)-0} + d_s\rho' [\frac{U_d^{\cdot}(t)}{g} + 1]\, ga(t) \cos\theta + \\ + \frac{a(t)\, a^{\cdot}(t)}{\epsilon k} \ln\frac{a(t)}{L(t)} + p_{f\infty}(t) \tag{5.3}$$

By expanding the solid-phase pressure jump, derived from Eqs (5.2) and (5.3), on the cluster surface in the neighbourhood of front points by the Davies–Taylor procedure we obtain the following system of differential equations relating the inhomogeneity velocity and its size

$$a^{\cdot 2}(t)(1-\lambda)\, [\frac{1+\lambda}{2} + \ln\frac{a(t)}{L(t)} - \frac{a(t)\, L^{\cdot}(t)}{a^{\cdot}(t)\, L(t)}] + \\ + (1-\lambda)\, a(t)\, a^{\cdot\cdot}(t) \ln\frac{a(t)}{L(t)} + \frac{a(t)\, a^{\cdot}(t)}{\epsilon k \rho d_s} \ln\frac{a(t)}{L(t)} + \\ + \frac{7}{2} U_d^2(t) - \frac{p_{s\infty}}{d_s\rho} + \frac{p_0(\mu-\mu')^2}{\rho d_s(\mu+\mu')^2}\, [3 - 7\mu + \frac{4\mu'(1-\mu)}{\mu-\mu'}] = 0 \\ \pm 2U_d(t)\, a^{\cdot}(t) \pm (\lambda+1)\, U_d^{\cdot}(t)\, a(t) + 4U_d^2(t) \pm \\ \pm (\lambda-1)\, ga(t) - \frac{8\mu p_0 (\mu-\mu')^2}{\rho d_s(\mu+\mu')^2} = 0 \tag{5.4}$$

The function $L(t)$ is defined, as before [3], by the condition $U_d(t) \equiv 0$ for $\lambda = 1$. This condition means that there are no macroscopic motions of the solid phase in the quasi-homogeneous bed.

As before, we take the linear expression (3.8) as the approximation of the function $\mu(\rho)$, assuming $\mu \gg 1$, $r > a(t)$. Then from Eq. (5.4) we obtain

$$7U_d^2(t) - (1-\lambda)^2 a^{\cdot 2}(t) + \frac{2p_0(1-\lambda)[4\lambda - 8 + \mu(3\lambda - 7)]}{\rho d_s(1+\lambda)^2} = 0 \\ \pm(\lambda+1)\, U_d^{\cdot}(t)\, a(t) \pm 2U_d(t)\, a^{\cdot}(t) + 4U_d^2(t) \pm \\ \pm (\lambda-1)\, ga(t) - \frac{8p_0\mu(1-\lambda)^2}{\rho d_s(1+\lambda)^2} = 0 \tag{5.5}$$

The steady-state solution of system (5.5) has the form

$$a_* = \pm\frac{32}{7}\frac{p_0(\mu\lambda+2)}{\rho d_s g(1+\lambda)^2}, \quad U_{d*}^2 = \frac{2p_0(\lambda-1)[4\lambda - 8 + \mu(3\lambda-7)]}{7\rho d_s(1+\lambda)^2} \tag{5.6}$$

and has a physical meaning ($a_* \geqslant 0$, $U_{d*}^2 \geqslant 0$) only in the case when $\lambda \in [0, 1)$.

Therefore, without magnetic action on the bed in the plane problem no inhomogeneity has steady states [3], while when the field is applied they appear for rarefied clusters. In particular, the equilibrium size of the bubble due to the magnetic properties of the solid phase is $a_{*b} = 64p_0/(7\rho d_s g)$.

Eliminating $U_d(t)$, from Eqs (5.6) we obtain a differential equation describing the evolution of the size of plane cluster in the model considered

$$\ddot{a}(t) = \frac{2[(1-\lambda)\dot{a}^2(t) - \psi_1]}{(\lambda^2-1)a(t)} + \frac{\sqrt{7}}{(\lambda^2-1)a(t)\dot{a}(t)}[(1-\lambda)^2\dot{a}^2(t) +$$

$$+(\lambda-1)\psi_1]^{1/2}[\frac{4}{7}(1-\lambda)\dot{a}^2(t) - \frac{4}{7}\psi_1 \mp a(t) - \psi_2] \tag{5.7}$$

$$\psi_1(\mu,\lambda) = \frac{7[4\lambda - 8 + \mu(3\lambda-7)]}{32(1+\lambda)^2}, \quad \psi_2(\mu,\lambda) = \frac{7\mu(1-\lambda)}{8(1+\lambda)^2}$$

In Eq. (5.7) we changed to dimensionless variables using the scales of length $a_{*b} = 64p_0/(7\rho d_s g)$ and time $(a_{*b}/g)^{1/2}$.

The autonomous dynamical system corresponding to Eq. (5.7) has a discontinuity on the right-hand side on the horizontal axis $\dot{a} = 0$ of the phase plane $(a, \dot{a})$. In this case, the evolution of a cylindrical inhomogeneity is quite different from the evolution of a spherical cluster occupying a bounded region (cf. (4.8)). The boundary of allowable motions of the representative point in the plane is defined by the condition

$$(\lambda - 1)^2\dot{a}^2 + (\lambda - 1)\psi_1 \geqslant 0$$

This condition is satisfied for rarefied clusters $(0 \leqslant \lambda < 1,\ \psi_1 < 0)$ and also for dense clusters with a relative concentration of particles $\lambda \in [7/3, \rho^{-1}]$ (in this case, $\psi_1 \geqslant 0$). In the case $\lambda \in (1, 7/3)$ when $\psi_1 < 0$, there is a "forbidden zone" in the phase plane along the horizontal axis $\dot{a} = 0$. Its width is $\Delta_0 = 2\sqrt{(\psi_1/(1-\lambda))}$. All the phase trajectories of the evolution equation (5.7) lie outside this region. The presence of a forbidden zone means a lower bound on the absolute value of the rate $\dot{a}$ of the change in size of a not too dense cluster. Such inhomogeneities in a planar fluidized bed of magnetic particles cannot evolve more slowly than the boundaries of the forbidden zone prescribe.

As an example we shall consider a fluidized system with the parameters $\rho = 0.4$, $\mu(\rho = 0.4) = 10$ in three cases: $\lambda = 0$ (a bubble), $\lambda = 3/2 \in (1, 7/3)$, $\lambda = 29/12 \in [7/3, \rho^{-1}]$. The phase patterns of Eq. (5.7) for the types of inhomogeneities considered are shown qualitatively in Figs 4–6.

Bubbles and rarefied clusters. The most typical singularity of the phase plane is the presence of a critical point on the abscissa Oa, that is accounted for by the existence of the equilibrium size of such inhomogeneities (for a bubble $a_* = a_{*b} = 1$). The critical point is a saddle $(\dot{a} > 0)$ or a centre $(\dot{a} < 0)$, and the phase trajectories are normal to the Oa axis (Fig. 4). Consequently, the common segment Oa_* of the phase trajectories degenerates into a continuum of the stable equilibrium states of inhomogeneity. In this case, the representative point may stay in this continuum for an infinitely long time. In the general case, we know [15, 16], that similar segments at the splice boundary of different "sheets" of the phase space of dynamical systems with discontinuous right-hand sides are regions of stable "sliding motion".

The presence of such a stable region fundamentally distinguishes the dynamics of bubbles and rarefied clusters in a plane magnetic bed from their dynamics in non-magnetic particle systems [3]. It also denotes the stabilizing effect of the field on such inhomogeneities $(\lambda < 1,\ a < a_*)$ contrary to the effect of their smoothing present in the three-dimensional models ([7] and Secs 3 and 4).

The evolution of inhomogeneity depends on the position of the phase point $(a, \dot{a})$ at the initial instant of time. From everywhere in the region 1 (shown shaded in Fig. 4) the representative point is attracted to the stable set Oa_*, and the cluster size stabilizes. Its location in regions 2 or 3 means that the cluster mass increases with time as a result of particle influx from outside. The rate of this influx in the first case changes non-monotonically, passing through a minimum, and in the second case it increases

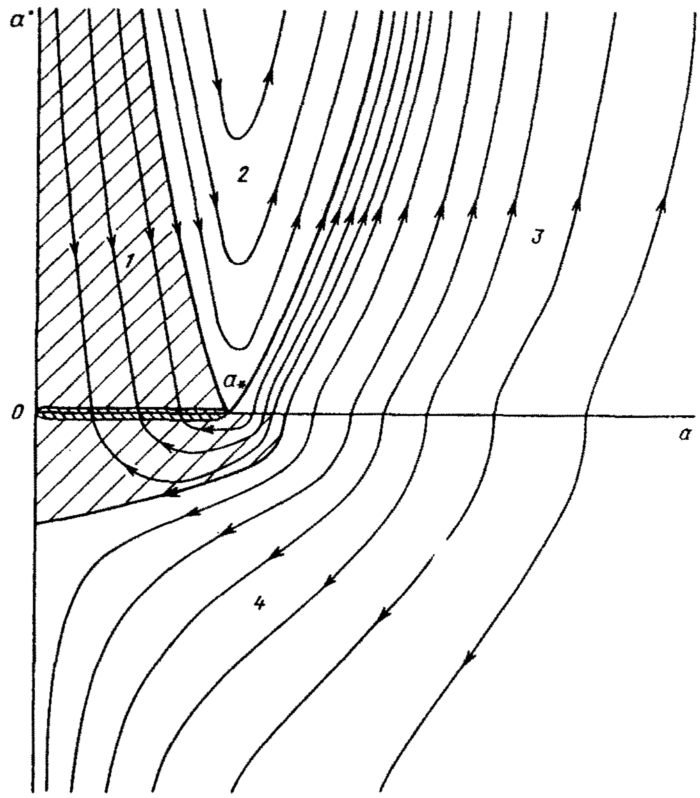

FIG. 4.

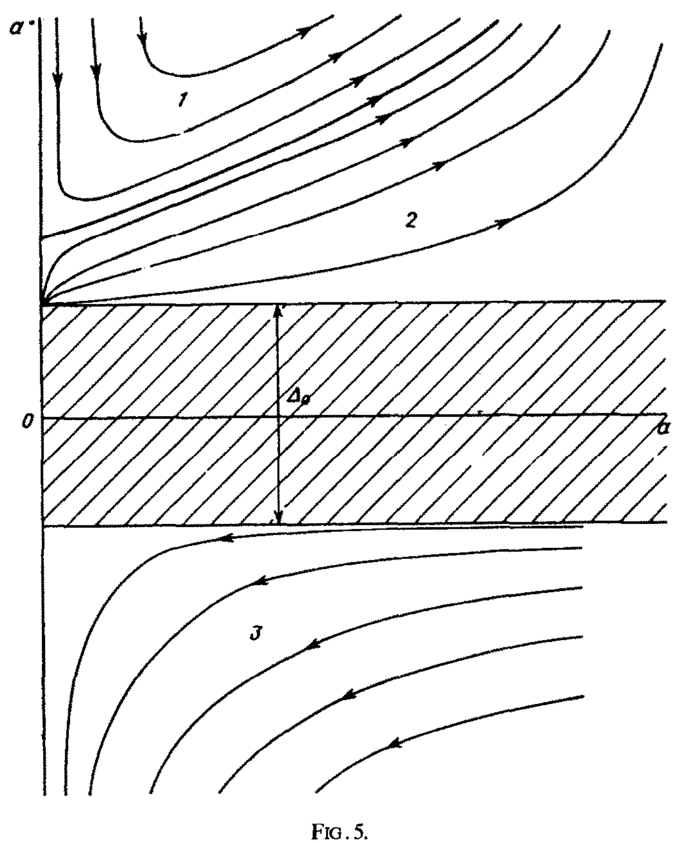

FIG. 5.

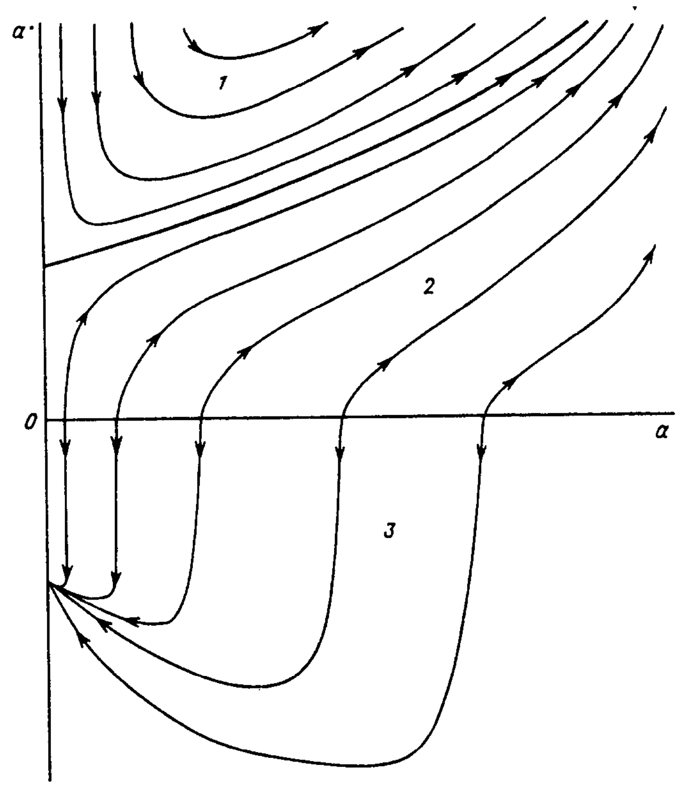

FIG. 6.

monotonically. Region 4 corresponds to cluster states wherein it loses its mass by returning particles to the external bed.

Thus, as in the case of non-magnetic particles, cluster states with sizes exceeding the critical size ($a > a_*$) are unstable. We note that already in the fields of moderate strength ($H \sim 10^2 - 10^3$ Oe) the stability region Oa_* covers practically the entire range of inhomogeneity sizes ($a_{*b} \sim 1$ m).

The dense cluster $\lambda \in (1, 7/3)$. The distinctive feature of the phase pattern of Eq. (5.7) is the presence of a forbidden zone limiting the domains of motion of the representative point in the phase plane (Fig. 5, the forbidden zone is shaded). The location of the phase point (a, $a^\bullet$) at the initial instant of time in regions 1 or 2, as in the previous case, corresponds to the condition for the inhomogeneity to grow at a monotonic (2) or non-monotonic (1) rate. A phase point located in region 3 represents by its motion the evolution of an inhomogeneity which loses mass at a monotonically increasing rate. The upper limit of the lifetime of these clusters can be approximately estimated using the relation $t_* = 2a_0/[\Delta_0(a_{*b}g)^{1/2}]$, where a_0 is the initial radius of the cluster.

The dense cluster $\lambda \in [7/3, \ \rho^{-1}]$. In this case, the pattern of the increase in the inhomogeneity is the same as in the two cases considered above (Fig. 6, regions 1 and 2). The process of the dissolution of a cluster acquires a new feature: the velocity of the cluster boundary at the instant it disappears is the same for inhomogeneities of any relative density $\lambda \in [7/3, \ \rho^{-1}]$. The rate of dissolution increases to a maximum and then decreases to the specified constant value at $a = 0$ (region 3 in Fig. 6).

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